MHD Waves

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What are MHD waves?

• How do we communicate in MHD?  MHD is kind!

• MHD waves are propagating perturbations of magnetic field, plasma velocity and plasma mass density, described by the MHD (single fluid approximation) set of equations, which connects the magnetic field $B$, plasma velocity $v$, kinetic pressure $p$ and density $\rho$.

• Non-relativistic approximation
Do we “expect” solar MHD waves?

- Corona has *frozen in field condition*
- Magnetic field rooted into turbulent photosphere
- Generates “waves” that dump energy in corona
- Alfvén/(Slow/Fast) Magneto-acoustic waves
Do we see MHD waves?

SOHO/TRACE examples (mainly TR and higher)

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Do we see MHD waves? - Moreton waves

1997/11/04 05:40:01 (UT)

1997/11/04 05:45:00

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Do we see MHD waves? – Sun quakes

Solar quakes: high energy electrons slam the solar surface

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Do we see MHD waves?

- Example: Rapid (every 15 s) TRACE 171 Angstrom image
- Track changes in brightness
- Wave travels outwards from B to T
Do we see MHD waves?

- Difference image in brightness out from the loop base
Linear theory of MHD waves

• Static/steady stationary background
• Superimpose linear motions on this background
• Write physical quantities as
  \[ f(r,t) = f_0(r) + f_1(r,t); \quad |f_1|/|f_0| << 1 \]
• Reduce full set of nonlin PDEs of MHD to a set of ODEs
• Choice: initial value problem, boundary value problem, eigenvalue problem
• Eigenvalue problem of linear waves/oscillations: \( \exp(i\omega t) \)
**Linear theory of MHD waves**

**Linearised ideal MHD equations**

\[
\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot V_1 = 0
\]

\[
\rho_0 \frac{\partial V_1}{\partial t} = -\nabla \left( p_1 + \frac{B_0 \cdot B_1}{\mu} \right) + \frac{(B_0 \cdot \nabla)B_1}{\mu}
\]

\[
\frac{\partial B_1}{\partial t} = \nabla \times (V_1 \times B_0)
\]

\[
p_1 = c_s^2 \rho_1, \quad c_s^2 = \gamma p_0 / \rho_0
\]
Linear MHD waves in uniform plasma

- No characteristic length scale defined by the equilibrium
- Constant equilibrium magnetic field, e.g.

$$B_0 = B_0 \sin \alpha x_0 + B_0 \cos \alpha y_0$$

- Superposition of linear waves

$$\exp(ik_xx + ik_yy + ik_zz), \ k = (k_x, k_y, k_z) = \text{wave vector}$$
Linear MHD waves in uniform plasma

Characteristics speeds:
- Alfvén speed: \( v_A = B_0 (4\pi \rho_0)^{-1/2} \)
- Sound speed: \( c_s = (\gamma \rho_0 / \rho_0)^{1/2} \)
Consider dynamics of perturbations of this stationary state. In the linear limit, the set of MHD equation splits into two uncoupled subsets:
Consider dynamics of perturbations of this stationary state. In the linear limit, the set of MHD equations splits into two uncoupled subsets:

(i) for the variables $V_y$ and $B_y$ (Alfvén wave)

(ii) and for $\rho, p, V_x, V_z$ and $B_x$ (magnetoacoustic waves)
Linear MHD waves in uniform plasma

Alfvén waves

\[
\left( \frac{\partial^2}{\partial t^2} - v_{Az}^2 \frac{\partial^2}{\partial z^2} \right) V_y = 0
\]

Properties:

(i) Transverse oscillation driven by

(ii) Incompressible (in linear limit)

(iii) Can't propagate across field lines

(iv) Group velocity \((\delta \omega / \delta k)\) is along \(B_0\)

\[
v_{Az} = \frac{B_0 \cos \alpha}{(4\pi \rho_0)^{1/2}}
\]
Linear MHD waves in uniform plasma

Alfvén waves

\[
\left( \frac{\partial^2}{\partial t^2} - v_{Az}^2 \frac{\partial^2}{\partial z^2} \right) V_y = 0 \quad \quad v_{Az} = B_0 \cos \alpha / (4 \pi \rho_0)^{1/2}
\]

Properties:

(i) Transverse oscillation driven by magnetic tension forces

(ii) Does not perturb density

(iii) Can’t propagate across field lines

(iv) Group velocity \((\delta \omega / \delta k)\) is along \(B_0\)
Linear MHD waves in uniform plasma

Alfvén waves

\[
\left( \frac{\partial^2}{\partial t^2} - v^2_{Az} \frac{\partial^2}{\partial z^2} \right) V_y = 0 \quad v_{Az} = B_0 \cos \alpha / (4\pi \rho_0)^{1/2}
\]

Properties:

(i) Transverse oscillation driven by magnetic tension forces

(ii) Does not perturb density \( \Rightarrow \) incompressible (in linear limit)

(iii) Propagate across field lines

(iv) Group velocity \((\delta \omega / \delta k)\) is along \(B_0\)
Linear MHD waves in uniform plasma

Alfvén waves

\[
\left( \frac{\partial^2}{\partial t^2} - \nu_{Az}^2 \frac{\partial^2}{\partial z^2} \right) V_y = 0 \quad \nu_{Az} = \frac{B_0 \cos \alpha}{(4\pi \rho_0)^{1/2}}
\]

Properties:

(i) Transverse oscillation driven by magnetic tension forces

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(iv) Group velocity \( (\delta \omega/\delta k) \) is along \( B_0 \)
Linear MHD waves in uniform plasma

Alfvén waves

When \( \mathbf{B}_0 \parallel z_0 \) there can be two \textit{linearly polarized} plane Alfvén waves, one perturbing \( V_y, B_y \) and the other \( V_x, B_x \).

For harmonic perturbations \([\exp(i\omega t - k z)]\) combination of two linearly polarized waves gives us \textit{elliptically polarized} Alfvén waves:

\[
B_y = A \cos(\omega t - k z), \quad A, B = \text{const}
\]
\[
B_x = B \sin(\omega t - k z),
\]

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Alfvén waves

The vector of the magnetic field perturbation rotates along an ellipse at the \(x,y\)-plane.

When \(A = B\), the wave is *circularly polarized*, with \(|B| = \text{const.} \)

Circularly polarized Alfvén waves (even of finite amplitude) are an *exact* solution of the ideal MHD equations for a uniform plasma.
Linear MHD waves in uniform plasma

Magnetoacoustic waves

\[
\left( \frac{\partial^2}{\partial t^2} - v^2_A \frac{\partial^2}{\partial z^2} \right) \left( \frac{\partial^2}{\partial t^2} - v^2_s \frac{\partial^2}{\partial z^2} \right) - v^2_{Ax} \left( \frac{\partial^4}{\partial t^2 \partial z^2} \right) V_z = 0, \quad v_{Ax} = B_0 \sin \alpha / (4 \pi \rho_0)^{1/2}
\]

Harmonic perturbations: \( V_z \sim \exp[i(\omega t - k z)] \)

Dispersion relation for MAW:

\[
(\omega^2 - v_A^2 \cos^2 \alpha k^2)(\omega^2 - v_s^2 k^2) - v_A^2 \sin^2 \alpha \omega^2 k^2 = 0
\]

DR bi-quadratic \( \rightarrow \) slow and fast magnetoacoustic waves
Linear MHD waves in uniform plasma

Magnetoacoustic waves

The polar plot for phase speeds $\left(\omega/k\right)$ for case $\beta < 1$
Linear MHD waves in uniform plasma

Magnetoacoustic waves

Here \( \tilde{\beta} = 0.36 < 0.5 \)

Here \( \tilde{\beta} = 0.81 > 0.5 \)

Here \( \tilde{\beta} = 1 \)

\[ C_f^2 = C_S^2 + V_A^2; \quad C_m = \min(C_S, V_A); \quad C_M = \max(C_S, V_A); \quad \tilde{\beta} = \frac{C_m^2}{C_M^2} \]
Linear MHD waves in uniform plasma

Group velocity

Initial perturbation corresponding either Alfven or slow or fast magnetosonic wave in the form of wave packet:

\[ f_1 = \Im \{ \Phi_0(r) \exp (i k_0 \cdot r) \}, \quad L k_0 << 1 \quad (2.13) \]

Fourier expansion:

\[ \Phi_0(r) \exp (i k_0 \cdot r) = \int \hat{\Phi}_0(k) \exp [i(k + k_0) \cdot r] dk \quad (2.14) \]

Contribution from \( k : \quad |k| << |k_0| \).
Look for solution in the form  

\[ f_1(\mathbf{r}, t) = \Re\{F(\mathbf{r}, t)\} \]

Each Fourier component has frequency  

\[ \omega(\mathbf{k}) \Rightarrow \]

\[ F(\mathbf{r}, t) = \int \hat{\Phi}_0(\mathbf{k}) \exp\left[i(\mathbf{k} + \mathbf{k}_0) \cdot \mathbf{r} - i\omega(\mathbf{k} + \mathbf{k}_0)t\right]d\mathbf{k} \tag{2.15} \]

\[ |\mathbf{k}| \ll |\mathbf{k}_0| \Rightarrow \omega \approx \omega_0 + \mathbf{V}_g \cdot \mathbf{k}, \quad \mathbf{V}_g \text{ is group velocity} \]

\[ \omega_0 = \omega(\mathbf{k}_0), \quad \mathbf{V}_g = \left. \frac{\partial \omega}{\partial \mathbf{k}} \right|_{\mathbf{k}=\mathbf{k}_0} \tag{2.16} \]

\[ F(\mathbf{r}, t) = \exp\left[i(\mathbf{k} \cdot \mathbf{r} - \omega_0 t)\right] \int \hat{\Phi}_0(\mathbf{k}) \exp\left[i\mathbf{k} \cdot (\mathbf{r} - \mathbf{V}_g t)\right]d\mathbf{k} \]

\[ = \Phi_0(\mathbf{r} - \mathbf{V}_g t) \exp\left[i(\mathbf{k}_0 \cdot \mathbf{r} - \omega_0 t)\right] \tag{2.17} \]

Envelope propagates with group velocity  \[ \mathbf{V}_g \]
For Alfven waves: \[ V_{gA} = V_A \frac{\mathbf{B}_0}{B_0} = V_A \hat{z} \] (2.18)

For magneto acoustic waves:

\[
V_{g\pm} = \frac{C^4 \mathbf{k} - C_S^2 V_A^2 k \hat{z} \cos \varphi}{C_{\pm} k [2 C^2_{\pm} - (C_S^2 + V_A^2)]} \] (2.19)

Curve \[ V_g (\varphi) \ (0 \leq \varphi < 2\pi) \]

is Fridrichs diagram

Direct calculation shows that

\[ \mathbf{k} \cdot \frac{\partial V_g}{\partial \varphi} = 0 \Rightarrow \]

rule for finding \[ V_g (\mathbf{k}) \]
Magnetooacoustic waves

The polar plot for group speeds \((d\omega/dk)\)
Properties:

(i) Anisotropic wave propagation largely confined to magnetic field

(ii) Driven by magnetic pressure and tension forces

(iii) Does perturb density/pressure

(iv) Can't propagate across field lines
Linear MHD waves in uniform plasma

Slow waves

Properties:

(i) Anisotropic wave propagation largely confined to magnetic field

(ii) Driven by magnetic pressure and tension forces

(iii) Does perturb density/pressure

(iv) Can't propagate across field lines

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**Linear MHD waves in uniform plasma**

**Slow waves**

Properties:

(i) *Anisotropic wave propagation* largely confined to magnetic field

(ii) Driven by magnetic pressure *and* tension forces

(iii) Does *perturb* density/pressure

(iv) Can’t propagate across field lines
Fast waves

Properties:

(i) Roughly isotropic wave propagation
(ii) Driven by magnetic pressure and tension forces
(iii) Does perturb density/pressure
(iv) Propagates fastest perpendicular to $B$
Linear MHD waves in non-uniform plasma

- Characteristic length scale defined by the inhomogeneity
- Equilibrium quantities are functions of position
- Continuum of resonant Alfvén and slow waves
- Discrete slow and fast modes; discrete Alfvén modes
- Efficient damping in non-ideal MHD
- MHD waves with mixed character and wave transformation
Linear theory of MHD waves

• Original MHD theory by e.g., Edwin & Roberts (1983) modelled a flux tube as a magnetic cylinder.

• It was found that there are many different types of MHD waves, e.g., fast/slow magneto-acoustic modes in magnetically twisted cylinder (Erdélyi & Fedun 2010).
Linear MHD waves in non-uniform plasma

- Properties of MHD waves depend upon the angle between the wave vector and the magnetic field. Waves "feel" the direction of the field.

- When the magnetic field is not straight, Alfvén and slow waves should follow the field, because they are confined to the field.

- Even when the field is straight, inhomogeneities in the field absolute value, density and pressure affect the characteristic speeds of the waves (the Alfvén and the sound speeds) and, consequently, affect the waves.

- Guided propagation of MHD waves, linear coupling of different MHD modes, phase mixing of Alfvén waves, resonant absorption, appearance of wave dispersion, etc.
Linear MHD waves in non-uniform plasma

- $v_A(x)$
- $v_s(x)$
- $\rho_0(x)$
- $B_0$
- $k$

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Linear MHD waves in non-uniform plasma

Total pressure balance: \( p_{\text{total}}(x) = p_0(x) + \frac{B_0^2(x)}{8\pi} = \text{const.} \)

Characteristic speeds:

- **Alfvén speed** \( v_A(x) = \frac{B_0(x)}{[4\pi \rho_0(x)]^{1/2}} \),
- **Sound speed** \( v_s(x) = [\gamma p_0(x)/\rho_0(x)]^{1/2} \),
- **Tube (cusp) speed** \( v_T(x) = \frac{v_s v_A}{[v_s^2 + v_A^2]^{1/2}} < v_s, v_A \).
Linear MHD waves in non-uniform plasma

Fourier transform in homogeneous directions \((y, z)\)

\[
f(x) \exp[i(\omega t - k_y y - k_z z)],
\]

Boundary conditions at fixed \(x\) \(\Rightarrow\) Dispersion Relation

\[
D(\omega, k_y, k_z, [B_0(x), \rho_0(x) \text{ and } p_0(x)]) = 0.
\]
Linear MHD waves in non-uniform plasma

Consider: $\delta/\delta y = 0$, though $V_y, B_y \neq 0$ (i.e. 2.5D)

Alfvén modes
[perturbing $V_y, B_y$]

Magnetoacoustic modes
[perturbing $V_x, V_z, B_x, B_z, \rho$]

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Magnetoacoustic modes are governed by:

\[
\frac{d}{dx} \left( \frac{\Lambda(x)}{m_0^2(x)} \frac{dV_x}{dx} \right) - \Lambda(x)V_x = 0,
\]

\[\Lambda(x) = \rho_0(x)[\omega^2 - k_z^2v_A^2(x)], \quad m_0^2(x) = \frac{(k_z^2v_s^2 - \omega^2)(k_z^2v_A^2 - \omega^2)}{(v_s^2 + v_A^2)(k_z^2v_T^2 - \omega^2)},\]

+ B.C.s=eigenvalue problem. Eigenfunctions define transversal (x) structure of waves; eigenvalues define dispersion for waves.

Singualarities: \{ \begin{align*}
\text{Alfvén} & \quad \frac{\omega}{k_z} = v_A(x) \\
\text{Cusp} & \quad \frac{\omega}{k_z} = v_T(x)
\end{align*} \] resonances!
Magnetoacoustic modes

Evanescent solutions: *modes* or *trapped* or *guided* (or *ducted*) waves; Dispersion is determined by the ratio of the longitudinal wavelength to the characteristic spatial scale of inhomogeneity.

The modes can have different structures in $x$ direction (inhomogeneity), which allows us to classify them:

- *kink* and *sausage* modes (perturbing or not perturbing the structure axis, respectively)

- *body* and *surface* modes (oscillating or evanescent inside the structure, respectively, and both evanescent outside the structure)
Kelvin-Helmholtz (KH) and Resonant Flow (RF) instability

Magnetosheath – magnetopause - magnetosphere
Heliopause – interstellar wind
Slow/fast solar wind boundary layer
Sunspots/Coronal plums
Helioseismology

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Kelvin-Helmholtz (KH) and Resonant Flow (RF) instability

Magnetosheath – magnetopause – magnetosphere

Generation of Pc5 waves, energisation of magnetosphere

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Kelvin-Helmholtz (KH) and Resonant Flow (RF) instability

Magnetosheath – magnetopause – magnetosphere
Heliopause – interstellar wind
Slow/fast solar wind boundary layer
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Helioseismology
Running penumbral wave generation by RFI
Kelvin-Helmholtz (KH) and Resonant Flow (RF) instability

Magnetosheath – magnetopause – magnetosphere
Heliopause – interstellar wind
Slow/fast solar wind boundary layer
Sunspots/Coronal plums

Helioseismology???
Kelvin-Helmholtz Instability (KHI)

Unperturbed state:

for $y < 0$:

$\rho = \rho_{01}, \mathbf{B} = B_{01} \hat{x}, \mathbf{V} = U_{01} \hat{x}$

for $y > 0$:

$\rho = \rho_{02}, \mathbf{B} = B_{02} \hat{x}, \mathbf{V} = U_{02} \hat{x}$

$C_S >> V_A \Rightarrow \text{Alfvenic perturbations can be considered as incompressible: } \rho = \text{const} \Rightarrow \nabla \cdot \mathbf{V} = 0$
Linearized equations: \( f(t, x, y) = f_0 + \tilde{f}(t, x, y) \)

\[
\nabla \cdot \tilde{V} = 0 \quad (1)
\]

\[
\rho_0 \left( \frac{\partial \tilde{V}}{\partial t} + (V_0 \cdot \nabla)\tilde{V} \right) = -\nabla \left( \tilde{p} + \frac{B_0 \cdot \tilde{B}}{\mu} \right) + \frac{(B_0 \cdot \nabla)\tilde{B}}{\mu} \quad (2)
\]

\[
\frac{\partial \tilde{B}}{\partial t} = \nabla \times (\tilde{V} \times B_0 + V_0 \times \tilde{B}) \quad (3)
\]

Boundary conditions: continuity of total pressure

\[
\tilde{p}_1 + B_{01} \cdot \tilde{B}_1 / \mu = \tilde{p}_2 + B_{02} \cdot \tilde{B}_2 / \mu \quad \text{at} \quad y = 0 \quad (4)
\]

Equation of perturbed interface \( y = \eta(t, x) \Rightarrow \)

Kinematic boundary condition (\( \tilde{V} = (u, v, 0) \)):

\[
\nu_{1,2} = \frac{\partial \eta}{\partial t} + U_{01,2} \frac{\partial \eta}{\partial x} \quad \text{at} \quad y = 0 \quad (5)
\]
\[ f_1 \propto \exp[i(k \cdot x - \omega t)] \Rightarrow \]
\[
\frac{dv}{dy} + iku = 0 \quad (6)
\]
\[
\rho_0(\omega - kU_0)u = k\tilde{p} \quad (7)
\]
\[
\rho_0(\omega - kU_0)v = -i \frac{\partial}{\partial y} \left( \tilde{p} + \frac{B_0\tilde{B}_x}{\mu} \right) - \frac{kB_0}{\mu} \tilde{B}_y \quad (8)
\]
\[
\omega\tilde{B}_x = i \frac{\partial}{\partial y} \left( U_0\tilde{B}_y - B_0v \right) \quad (9)
\]
\[
(\omega - kU_0)\tilde{B}_y = -kB_0v \quad (10)
\]
Kinematic boundary condition at \( y = 0 \):

\[
v_{1,2} = -i(\omega - kU_{0,1,2})\eta \quad \Rightarrow \quad \frac{v_1}{\omega - kU_{01}} = \frac{v_2}{\omega - kU_{02}}
\] (11)

Eliminate \( u \) and \( \vec{B}_y \) from (6)-(10)  

\[
\frac{d^2v}{dy^2} - k^2v = 0
\] (12)

\[
\tilde{p} + \frac{B_0\tilde{B}_x}{\mu} = i\rho_0[(\omega - kU_0)^2 - V_A^2k^2] \frac{dv}{dy}
\] (13)

\( v \to 0 \) as \( |y| \to \infty \)  

\[
v = \begin{cases} 
A_1e^{\mid k \mid y}, & y < 0 \\
A_2e^{-\mid k \mid y}, & y > 0 
\end{cases}
\] (14)
Substitute (13) and (14) in boundary conditions (4) and (11) \[ \Rightarrow \]

\[
\frac{A_1}{\omega - kU_{01}} - \frac{A_2}{\omega - kU_{02}} = 0 \quad (15)
\]

\[
\rho_{01}\left[\frac{(\omega - kU_{01})^2 - V_{A1}^2 k^2}{k^2(\omega - kU_{01})}\right]A_1 + \rho_{02}\left[\frac{(\omega - kU_{02})^2 - V_{A2}^2 k^2}{k^2(\omega - kU_{02})}\right]A_2 = 0 \quad (16)
\]

Linear system with respect to \( A_1 \) and \( A_2 \). There is non-trivial solution \[ \Rightarrow \] determinant is zero \[ \Rightarrow \] dispersion equation:

\[
(\rho_{01} + \rho_{02})\omega^2 - 2k(\rho_{01}U_{01} + \rho_{02}U_{02})\omega
\]

\[
+ k^2\left[\rho_{01}(U_{01}^2 - V_{A1}^2) + \rho_{02}(U_{02}^2 - V_{A2}^2)\right] = 0 \quad (17)
\]
\[ \omega = k \frac{\rho_{01} U_{01} + \rho_{02} U_{02}}{\rho_{01} + \rho_{02}} \pm \sqrt{\rho_{01} \rho_{02} [U_{KH}^2 - (U_{01} - U_{02})^2]} \] 

(18)

\[ U_{KH}^2 = \frac{(\rho_{01} + \rho_{02})(\rho_{01} V_{A1}^2 + \rho_{02} V_{A1}^2)}{\rho_{01} \rho_{02}} \] 

(19)

\[ |U_{01} - U_{02}| < U_{KH} \Rightarrow \Im(\omega) = 0 \Rightarrow \text{Stability} \]

\[ |U_{01} - U_{02}| > U_{KH} \Rightarrow \Im(\omega) \neq 0 \Rightarrow \text{Instability, increment } \propto k \]

Particular case:

\[ \rho_{01} = \rho_{02}, \quad B_{01} = B_{02} \Rightarrow V_{A1} = V_{A2} = V_A \Rightarrow U_{KH} = 2V_A \]
Model equilibrium state

Steady equilibrium state
Kelvin-Helmholtz instability (KHI)

Fast modes \((a, \theta=25^\circ; b, \theta=75^\circ); \beta=0\) (cold plasma); \(L=0\)

Forwards & backwards propagation, KHI